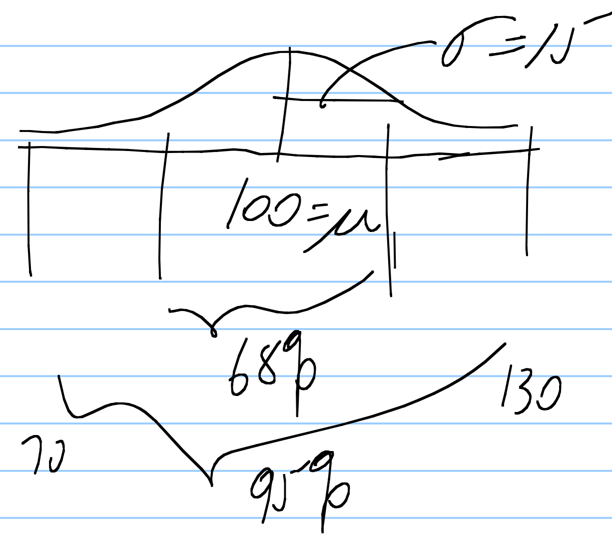
STT 200 7-16-10 12:40

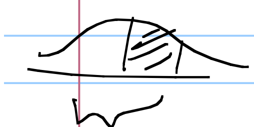
Note Title 7/16/2010

NORMAL DISTRIBUTIONS, Z-TABLE pg 210,
(ALL NORMALS ARE ALIKE IN σ UNITS FROM THE MEAN).
STANDARD SCORES.

IQ NORMAL MEAN 100 SD σ = 15.



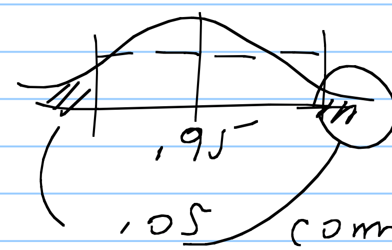
S.O. $P(IQ \text{ IN } 85 \text{ TO } 115) \sim .68$



$P(IQ \text{ IN } 100 \text{ TO } 115) \sim .68/2 = .34$

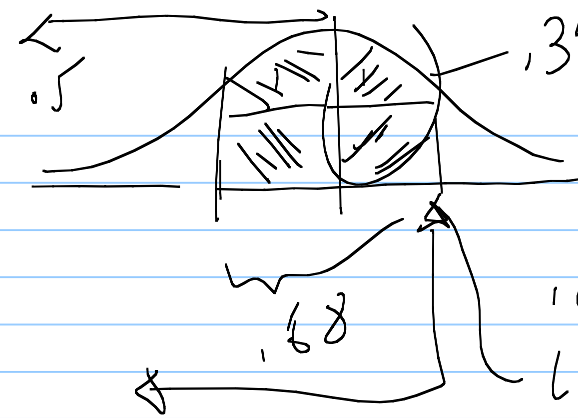
$P(IQ \text{ IN } 70 \text{ TO } 130) \sim .95$ (RULE OF THUMB)

$P(IQ > 130) \sim .025$



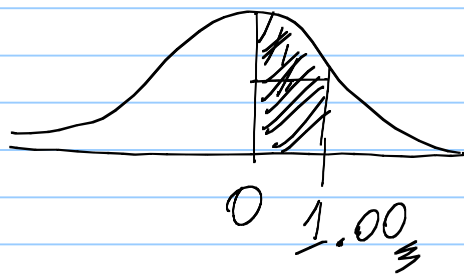
SYMMETRIC
WHOLE .8

.5



.84
LEFT OF

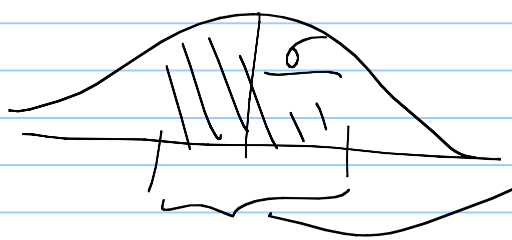
TABLE -



$\mu = 0$ $\sigma = 1$ STANDARD NORMAL

z	.00
1.0	$\boxed{.3413}$

68% SHOULD
REALLY BE 68.26%



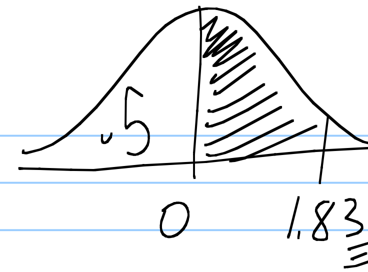
CLOSER TO .6826

Q. FIND $P(Z < 1.8\underline{3})$ (USE TABLE)

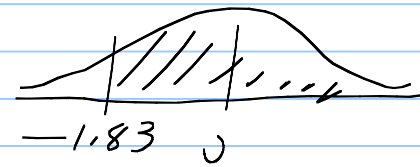
$= .5 + .4664 = .9664$

z $.0\underline{3}$

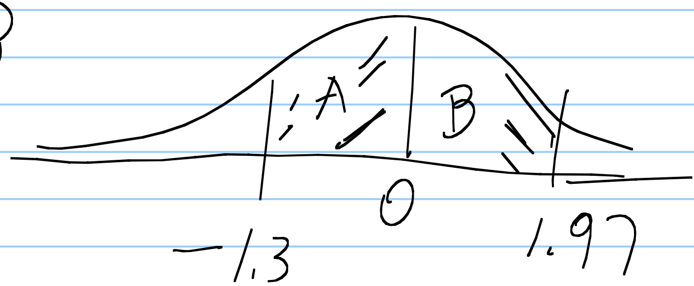
1.8 $.4664$

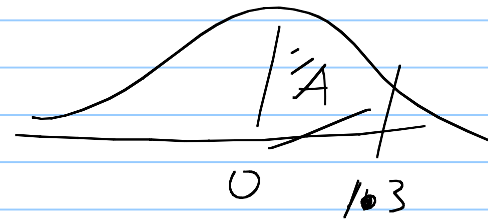


Q. $P(Z > -1.83)$



Q.





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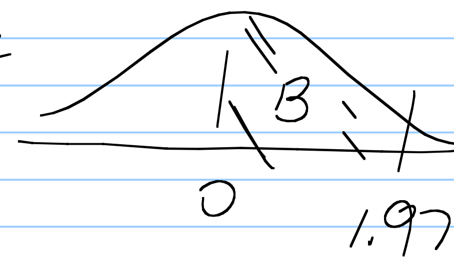

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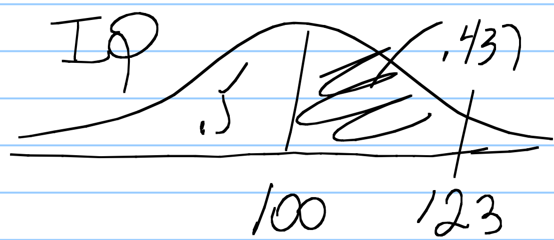
z-SCORES. IQ $\mu = 100$ $\sigma = 15$

$P(\text{IQ IN RANGE } 100 \text{ TO } 123) = P\left(Z \text{ IN } \frac{100-100}{15} \text{ TO } \frac{123-100}{15}\right)$

IQ 100 $\rightarrow$ $z = \frac{100-\mu}{\sigma} = \frac{100-100}{15} = 0$ RAW 123 $\rightarrow$ $z = \frac{123-100}{15} = 1.53$

SO $P(IQ \text{ IN } \underset{\mu}{100} \quad \underset{\mu + 1.53\sigma}{123}) = P(Z \text{ IN } 0 \text{ TO } 1.53)$



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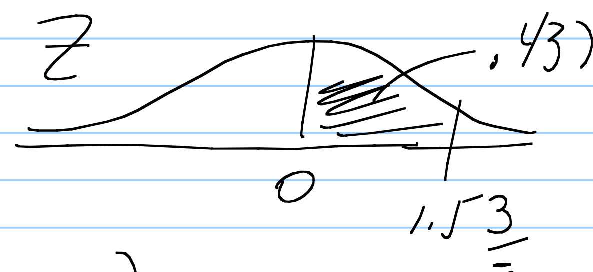


	.03
3	
1.5	$\boxed{.4370}$

SO TOO $P(IQ < 123) = .5 + .437 = .937$

SOMEONE w/ IQ = 123 IS ALMOST AT 94^{TH} PERCENTILE OF IQ.

NORMAL APPROX OF BINOMIAL

$EX = np = 100\left(\frac{1}{2}\right) = 50$

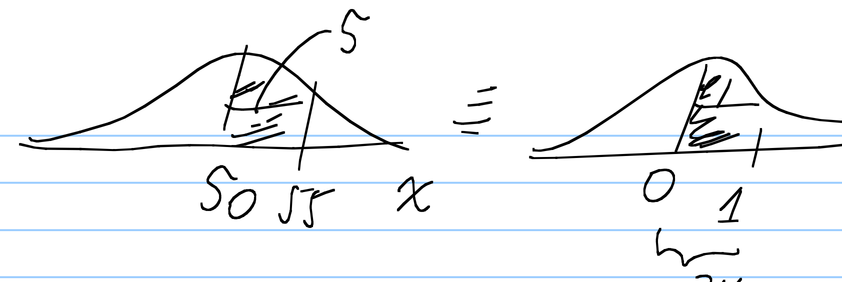
eg TOSS COIN 100 TIMES. $X = \#H$ IN 100 TOSSES

$\sigma_X = \sqrt{np(1-p)}$

$= \sqrt{100 \, \frac{1}{2} \, \frac{1}{2}} = 5$

NORMAL APPROX $\approx$

RAW $x = 55 \to z = \frac{55-50}{5} = 1$



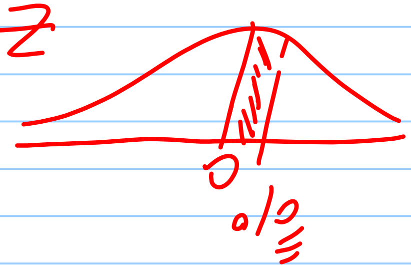
$P(\#H \text{ IN } 50 \text{ TO } \ldots 55)$

INCL INCL

$\approx P(\;49.5 \;\text{—}\; 55.5\;) = P\left(Z \text{ IN } \frac{49.5-50}{5} \text{ TO } \frac{55.5-50}{5}\right)$

$= P(Z \text{ IN } -0.10 \text{ TO } 1.10)$

Z



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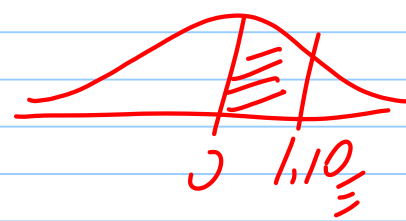
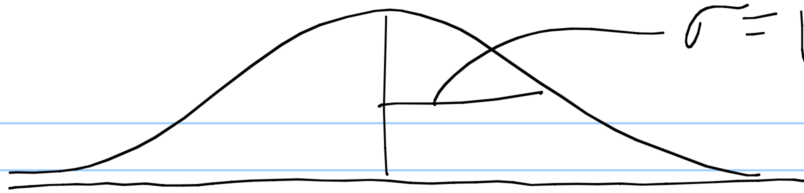


z	.00
.1	$\boxed{.039}$

z	.00
1.1	$\boxed{.3643}$

$P(\#H\ 50 \to 55) = .039 + .3643 = .4033$

BATTING $p = .3$ $n = 100$ $\mu = np = 30$ $\sigma = \sqrt{100 \cdot .3 \cdot .7}$



$\sigma = \sqrt{np(1-p)} = 4.58$

$\sigma = 4.58$ NOT SO FAR FROM 5

$\mu = np = 30$

$30 \pm 4.58 \approx 68\%$ INTERVAL

ACTUAL P(#HITS IN 27 → 34)

$\approx P\left(Z \text{ IN } \frac{26.5-30}{4.58} \text{ TO } \frac{34.5-30}{4.58}\right) \approx .61$

WHY DID I NO GO TO 26 → 34 - BLIPPED!

BINOMIAL n LARGE p SMALL

eg $n = 500 \quad p = \frac{1}{100} \qquad \mu = np = 500\,\frac{1}{100} = 5$

$$p(3) = \frac{500!}{3!\,497!}\left(\frac{1}{100}\right)^3\left(\frac{99}{100}\right)^{497}$$

$n = 5000 \quad p = \frac{1}{1000} \qquad \mu = np = 5000\,\frac{1}{1000} = 5$

$$p(3) = \frac{5000!}{3!\,4997!}\left(\frac{1}{1000}\right)^3\left(\frac{999}{1000}\right)^{4997}$$

$\sqrt{np(1-p)}$

$\sim \sqrt{np}$

VERY CLOSE TO ONE ANOTHER

AND $\sim e^{-5}\,\frac{5^3}{3!}$ $\qquad e \sim 2.718281828\ldots$

form

$$e^{-\mu}\,\frac{\mu^x}{x!} \quad x = 0, 1, 2 \ldots$$

POISSON

POISSON RAISIN COOKIES.

MAKE 144 COOKIES

$576 = 4(144)$

YOUR COOKIE

$p = 1/144 \qquad n = 576$

$np = 576\left(\frac{1}{144}\right) = 4$

$$p(3) \approx e^{-\mu}\frac{\mu^3}{3!} = e^{-4}\frac{4^3}{6}$$

$$= 0.1954$$

$3! = 3\cdot 2\cdot 1 = 6$

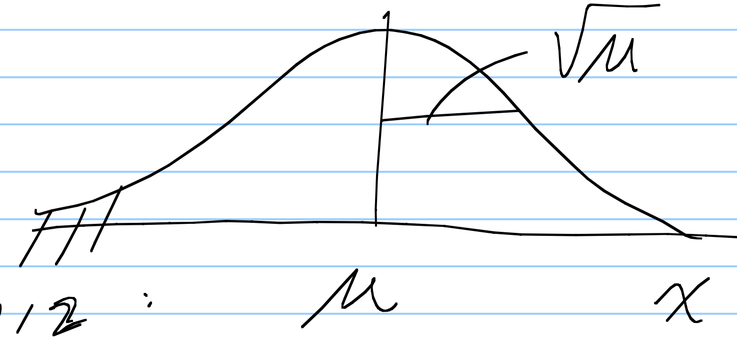
$$p(0) = e^{-4}\frac{4^0}{0!} = e^{-4} = 0.018$$

<u>COMMENT</u>: YOU MAY JUSTIFY THIS APPLICATION OF POISSON BY BAKING A LOT OF COOKIES, FINDING $\sim 2\%$ HAVE NO RAISINS, $\sim 20\%$ HAVE 3 RAISINS, ETC.

NORMAL APPROX OF POISSON.

POISSON
MEAN μ
(SD IS $\sigma = \sqrt{\mu}$)

$p(x) = e^{-\mu} \dfrac{\mu^{x}}{x!}$ $\approx$



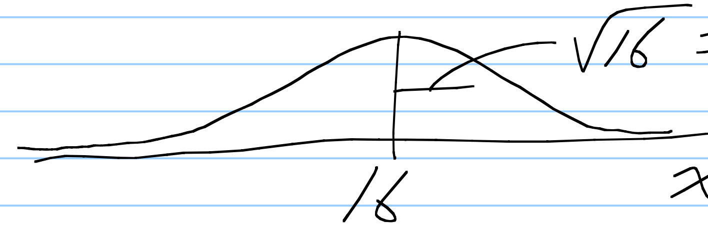
APPLICABLE (THIS COURSE) $\mu \geq 10$

SAY m+m Oatmeal Cookies (9" DIAMETER)

SAY AVG # m+m PER COOKIE IS 16

x = # OF m+m IN A GIVEN COOKIE.

$\approx$ DIST



$\sqrt{16} = \sqrt{\mu}$ FOR POISSON

x = # m+m

BEAUTIFUL! WE ONLY NEED μ!!!

eg WE AVG 25 CLAIMS / DAY $\sim$ DIST